Adaptive Diagonal Loading for Robust Minimum Power Distortionless Response Beamformer

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Abstract - **Most of the modern radar and sonar uses adaptive beam forming to eliminate the interference. The basic requirement of the adaptive algorithm is to eliminate the interference and increase the Signal-to-Interference-plus-Noise Ratio (SINR). The efficiency of the adaptive algorithm depends on the accurate estimation of the spatial covariance matrix. Generally the data used to estimate the covariance matrix should not contain any signal of interest, if so adaptive algorithm eliminates the signal also along with the interference. Diagonal loading is one of the most widely used and effective method to improve robustness of adaptive beamformer, but selecting the diagonal loading value has become a key aspect. In the absence of complete knowledge of signal characteristics and continuously changing environment, fixed diagonal loading method may not give desired performance, hence the adaptive method is essential. In this paper we propose adaptive diagonal loading method for Minimum Power Distortionless Response beamformer (MPDR), which systematically computes the diagonal loading value based on the covariance matrix and its eigen values. Simulation results demonstrates significant performance improvement of proposed method as compared with the fixed diagonal loading method.**

Keywords - **Spatial covariance matrix, Adaptive diagonal loading, Minimum power distortionless response beamformer.**

I. INTRODUCTION

The basic idea behind the adaptive algorithms is to eliminate unwanted signals from the side lobe regions. The adaptive algorithms are broadly classified into three categories [1] based on computing the adaptive weights. Recursive Least Squares method and Steepest Decent method converge much slower than that of Sample Matrix Inversion method. Sample Matrix Inversion method needs to estimate the sample covariance matrix from the available data. The data used to estimate the covariance matrix should not contain the desired signal, but in practical applications it is not possible to completely eliminate the desired signal.

Sufficient amount of data samples are required to estimate the spatial covariance matrix, otherwise beamformer performance degradation is expected[2]. If the data used to estimate the covariance matrix has the desired signal, then the adaptive algorithm places null in the direction of the signal of interest depending on the signal to noise ratio. Diagonal loading has proved as an effective way to improve the robustness of the optimal beamformer. But selecting the diagonal loading level is the problem of interest. Adaptive diagonal loading value has been estimated for steering vector errors by Vincent and Besson[2], DOA mismatch and array perturbations by Jiang, Zhu and Sun[3].

The structure of the paper as follows: explanation of adaptive diagonal loading method based on covariance matrix, data model used for the simulations, simulation results and finally conclusion along with references.

II. PROBLEM FORMULATION

1. Array model

Consider P narrow band signal sources impinging on the array of N elements, The complex envelope of the received vector can be written as

$$X(t) = A(\theta_p)s_p(t) + n(t) \dots\dots\dots\dots\dots (1)$$

where $X(t) = [x_1(t), x_2(t), x_3(t) \dots\dots\ x_N(t)]$, the complex envelope of the narrowband signals $s(t) = [s_1(t), s_2(t), s_3(t) \dots\ s_P(t)]$ and a complex white Gaussian noise, $n(t) = [n_1(t), n_2(t), n_3(t) \dots\ n_N(t)]$. $\boldsymbol{A}$ is a DOA matrix, $A = [a(\theta_0), a(\theta_1), a(\theta_2) \dots\ a(\theta_{P-1})]$.

If we add narrow band interferences, then

$$Y(t) = X(t) + I(\theta_p)j_p(t) \dots\dots\dots\dots\dots\dots (2)$$

Assuming that the noise is spatially and temporally uncorrelated, signals and interferences are statistically independent to each other but spatially and temporally correlated. The covariance matrix will be of the following form

$$R = \sigma_0^2 a(\theta_0)a^H(\theta_0) + \sum_{i=1}^{P-1} \sigma_i^2 I(\theta_i)I^H(\theta_i) + \sigma_n^2 \dots\dots (3)$$

The first term in (3) corresponds to the desired signal, the other term $\sum_{i=1}^{P-1} \sigma_i^2 I(\theta_i)I^H(\theta_i)$ corresponds to the interferences and σ_n^2 is the thermal noise power. In practical situations, R is replaced with the estimated spatial co-variance matrix by using K number of samples

$$\hat{R} = YY^H \dots\dots\dots\dots\dots\dots\dots (4)$$

2. MPDR with diagonal loading

The well known optimum beamformer minimizes the power under distortionless constraint

$$\begin{cases} \min\limits_{W} W^H R W \dots\dots\dots\dots\dots\dots\dots (5) \end{cases}$$

$$s.t\ W^H a(\theta_0) = 1$$

the solution for the above problem is

$$W_0 = \frac{R^{-1}a(\theta_0)}{a^H(\theta_0)R^{-1}a(\theta_0)} \dots\dots\dots\dots (6)$$

When R is replaced with $\hat{R}$, a with a_m and the data contains the desired signal MVDR becomes MPDR. For the diagonal loading $\hat{R}$ is replaced with $\hat{R} + \sigma_L^2 I$, where σ_L^2 is a diagonal loading value and I is identity matrix.

III. Proposed Adaptive Diagonal Loading Method

If we analyse the covariance matrix with R_{ratio}, where

$$R_{ratio} = \frac{mean\ of\ diag(R)}{mean\ of\ nondiag(R)} \dots\dots (7)$$

When there is no external signal then R_{ratio} value will be very high. When there is a external signal R_{ratio} will converge towards unity depends on the external signal power. We use External Signal Indicator (ESI) to indicate the presence of external signal. when ESI is enabled our proposed method is going to be applied otherwise conventional beamformer is used i.e. adaptive beamformer is performed only if the external interference is present.

Adaptive diagonal loading level can be derived from the eigen values and the non-diagonal mean of the spatial covariance matrix. The mean of eigen values of the covariance matrix corresponding to system noise is σ_n^2. finally the Load-to-Noise Ratio (LNR) is calculated.

$$LNR = 0.5 * (mean\ of\ nondiag(R\ in\ dB) - \sigma_n^2\ in\ dB) \dots\dots (8)$$

fig (1) shows that for various levels of system noise levels the ESI remains disabled since external signal is not present. In fig(2) we can observe the indication of external source by ESI for various levels of SINR. It can be observed that the R_{ratio} is crossing the threshold and enabling ESI. The threshold can be derived from the system noise.

Generally the eigen values of system noise will be less than the signal or interference, simple way of finding the eigen values corresponding to system noise is as follows: 1. Arrange the eigen values in ascending order, 2. Find out the boundary where eigen value is drastically changing, 3. Take the mean of all the eigen values before the boundary.

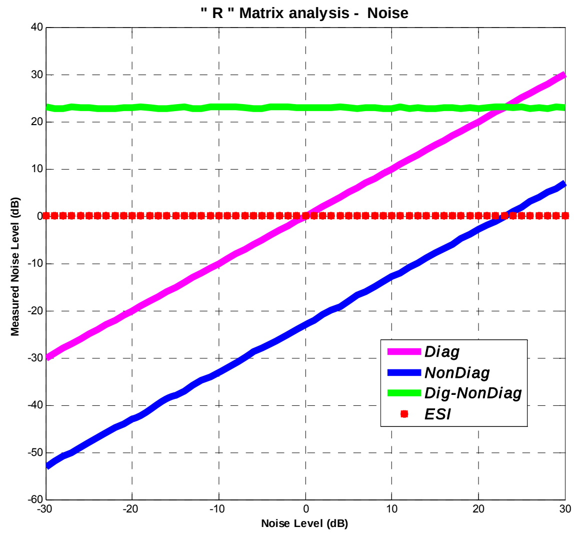

Fig. 1. Covariance Matrix analysis for Noise.

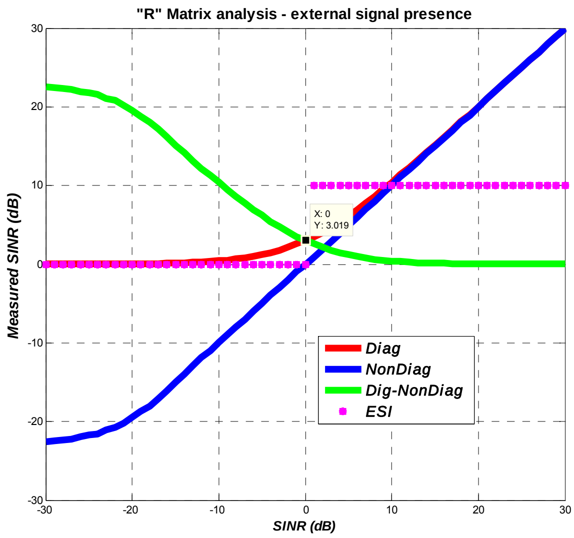

Fig. 2. Covariance Matrix analysis for external signal.

IV. Simulation Results

The above explained method has been validated with different scenarios. In the simulations we compare adaptive diagonal loading method with conventional beamformer, MPDR, fixed diagonal loading method.

Array used in this simulation has 25 element Uniform Linear Array with a half-wavelength inter element spacing. System noise is assumed as zero dB. Three narrow band interferences are injected with 25, 30, and 40 dB INR, at -38^0, -26^0, and $+31^0$ correspondingly. The desired signal is injected with a various levels of SNR at 0^0 of the antenna. The direction of the interferences are shown with red vertical lines.