

Spline Interpolated Adaptive Doppler Filter Based Velocity Estimation

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Abstract:

This paper discusses the application of cubic spline interpolation technique to predict acceleration (or velocity) and to adapt better window length for any range of target acceleration. The estimated velocity (or acceleration) is then smoothened out using Kalman and Adaptive Kalman filter. Simulated results have shown that in 'High-Level noise' scenario, 'spline interpolated adaptive filter' gives more accurate estimate in comparison to the existing method [1].

Keywords- Cubic spline, Signal-to-noise ratio Kalman filter and adaptive Kalman filter

I. INTRODUCTION

When signal is transmitted and received after being reflected from a moving target, the frequency of the signal changes, better known as Doppler effect [5]. Papic *et al*, [1], [6] estimated the Doppler shift through time dependent Fourier analysis, based on the rectangular window function. The rectangular window having an abrupt transition to zero outside the selected range, is known to introduce ripples. To alleviate the presence of large oscillations in both the pass band and the stop band one should use window function that ramps up and down towards zero gradually [2]. Hence Kaiser window can be used in place of rectangular window to obtain the finite sequence.

In [1], three consecutive peaks obtained from the spectrum are considered to minimize the error due to estimated frequency. The optimized frequency was obtained using simple interpolation procedure based on 'Second-order polynomial approximation'. Valarmathi *et al* [13], considered three consecutive peaks of the finite sequence obtained through Kaiser window [2], and used 'Lagrange's Polynomial interpolation' and 'Semi-Newton's iterative method' to determine Doppler Shift. It was observed from simulated results that the above methods did not proffer much improvement in estimation. In these methods the look-up table [1] was directly used for adapting window length. The look-up table provided window length for different SNRs at three constant accelerations, i.e 3, 7 and 15m/sec². This meant that other accelerations had to be approximated to these values in order to be able to select the window length. This approximation resulted in errors in estimation.

Based on the work by Papic *et al* [1], [6], Vladimir *et al* [3], Jubisa *et al* [4] and Hedrick [11], elucidates the importance of the length of window function. A short length window function helped avoid the effect of velocity averaging across a long interval but could result in bad velocity estimation and larger acceleration in case of intensive object dynamics. In case of low SNR, sufficient Window length is a must to contain relevant information on Doppler frequency shift. Tradeoff between echo signal SNR and target acceleration is done carefully to decide upon the window length. It was observed that the cubic Spline method of interpolation was more effective in adapting window length for determining acceleration. Further it was observed Spline interpolated Doppler filter with FFT gave better estimate of velocity in comparison to the complicated methods described earlier. Kalman filter [9], [10] was used to smoothen the estimated velocity and determine the acceleration of moving target.

This paper is organized in the following way. System description and Doppler frequency estimation are explained in section II. Window length adaptation technique are explained in section III. In section IV results of velocity estimation through these algorithm is simulated and analyzed. Finally the conclusion is given in section VI.

II SYSTEM DESCRIPTION

The block diagram for the velocity estimation is shown in the fig. 1. It consists of Doppler filter to find the Doppler shift from the echoed signal. Transmitted and received signals are synthesized based on the reference [13].

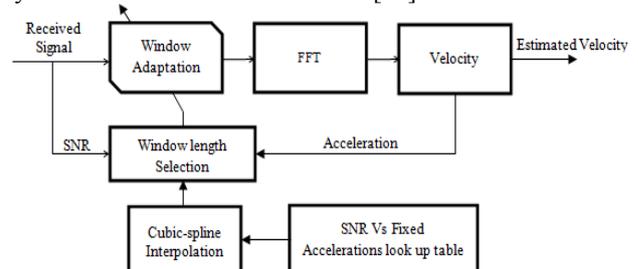


Fig. 1 Block diagram of velocity estimation

2.1 Windowing

Kaiser window [5] has a flexibility of choosing a desired side lobe level and the main lobe peak on varying the parameter (α). An added advantage of the Kaiser window is that its transition bandwidth is always small. Here the stop band attenuation (A_s) in dB assumed to be 25dB. Hence $\beta = 0.6$ approximately.

2.2 Fast Fourier Transform(FFT)

The windowed output is analyzed in frequency domain by performing the FFT [2] to get the peak frequency.

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}, \quad 0 \leq k \leq N-1 \quad (1)$$

A higher point FFT is applied to the window output so as to obtain a better display of the DFT $X(k)$. As the window size (M) would be less than the N , zeros are padded as they do not provide any additional information about the spectrum. The frequency corresponding to the highest magnitude of FFT is the received signal frequency f_i

2.1 Velocity Estimation

The Doppler frequency shift f_d is given as the frequency by which the received signal varies from the transmitted signal.

Hence, $f_d = f_1 \pm f_0$, where f_0 is the transmitted signal frequency. The velocity of the target is given by

$$v_p = f_d \times \frac{c}{f_0} \cos \theta \quad (2)$$

c - velocity of propagation = 3×10^8 m/s with $\cos \theta = \cos \theta_a \cos \theta_e$. The angles θ_e and θ_a are the elevation and azimuth angles. For the estimated Doppler frequency shift, target radial velocity v_r can be calculated as

$$v_r = \frac{f_d c}{2f_0} \quad (3)$$

2.2 Cubic-Spline Interpolation

Since the received signal has 'n' number of samples, proper samples should be selected through windowing technique to compromise the trade-off between the target maneuvering and the signal-to-noise ratio. For each observation, window length has to be adjusted based on the target acceleration and SNR. The look up table given in reference [1] provides the window length for different SNR's at constant three accelerations, namely 3 m/sec², 7 m/sec² and 15 m/sec². Other accelerations should be approximated to these values for selecting the window length. This approximation leads to errors in estimation. Hence interpolation technique was opted in the acceleration to adapt better window length. Cubic splines are curves used for the interpolation of data. They are constructed through a set of known data points. Here, corresponding to the SNR of the received signal, the accelerations and their respective window lengths obtained from the look up table given in reference [1], are interpolated using cubic spline interpolation method. In general, cubic spline interpolation has equations of two splines [4] where x is the acceleration and y is the window length

$$y = a_1(x-x_1)^3 + b_1(x-x_1)^2 + c_1(x-x_1) + d_1 \quad (4)$$

$$y = a_2(x-x_2)^3 + b_2(x-x_2)^2 + c_2(x-x_2) + d_2 \quad (5)$$

where $a_1, a_2, b_1, b_2, c_1, c_2,$ and d_1, d_2 are the constants. Substituting $x=x_1$ and $x=x_2$ in equations (4) and (5) gives,

$$d_1 = y_1; \quad d_2 = y_2$$

Taking the second derivative of equation (4) and (5)

$$y'' = 6a_1(x-x_1) + 2b_1 \quad (6)$$

$$y'' = 6a_2(x-x_2) + 2b_2 \quad (7)$$

considering $y''=y_1''$ at $x=x_1$ in equation (6) and $y''=y_2''$ at $x=x_2$ in equation (7) leads to

$$b_1 = y_1''/2 \quad \text{at} \quad b_2 = y_2''/2 \quad (8)$$

substituting equation (8) in (6), (7) and solving the same after replacing $y = y_1$ at $x=x_1$ in equation (6) and $y = y_2$ at $x=x_2$ in equation (7) to get the constants a_1 and a_2 as

$$a_1 = (y_2'' - y_1'')/6h_1; \quad a_2 = (y_3'' - y_2'')/6h_2; \quad (9)$$

where $h_1 = x_2 - x_1$ and $h_2 = x_3 - x_2$

Substituting all the constants and replacing $y=y_1$ at $x=x_1$ and $y=y_2$ at $x=x_2$ in equation (4), (5)

$$c_1 = (y_2 - y_1)/h_1 - y_2''h_1/6 - y_1''h_1/3 \quad (10)$$

Finally to find the unknown second derivatives, impose the compatibility condition that y_2' in spline 1 must equal y_2' in spline 2

$$3a_1(x_2-x_1)^2 + 2b_1(x_2-x_1) + c_1 = 3a_2(x_2-x_2)^2 + 2b_2(x_2-x_2) + c_2$$

On substitution of all constants and simplification leads to $h_1 y_1'' + 2(h_1 + h_2)y_2'' + h_2 y_3'' = 6[(y_3 - y_2)/h_2 - (y_2 - y_1)/h_1]$ (11)

In the second derivative of equations (4) and (5), if we consider the natural boundary conditions $y_1'' = 0$ and $y_3'' = 0$, we arrive at the following matrix equation

$$y_2'' = \frac{6[(y_3 - y_2)/h_2 - (y_2 - y_1)/h_1]}{2(h_1 + h_2)} \quad (12)$$

For the given window length corresponding to three accelerations values are taken as (x_1, y_1) , (x_2, y_2) and (x_3, y_3) and calculated the spline segment coefficients and plot the cubic spline. From the cubic spline plotted, the exact window length was obtained for a particular acceleration pertaining to particular SNR. to find was selected using which the optimized Doppler frequency shift f_d and eventually the velocity of the target v_p can be estimated.

2.3 Kalman filter

Here Kalman filter is used to smooth noisy measurement from the Doppler filter. This method has been already explained in [13] for estimating the velocity and acceleration of a moving target.

III WINDOW LENGTH ADAPTATION

The target may have different accelerations. As a result frequency also changes with time. Since we are considering fixed window length in Doppler and Kalman filters, it experiences a delay in tracking the target dynamics [1, 5, 9]. The applied window function length must be short enough to avoid the effect of velocity averaging across a long interval. This velocity averaging would result in bad velocity estimation in the case of intensive object dynamics resulting in larger acceleration. On the other hand window length has to be

long enough to contain relevant information on the Doppler frequency shift especially in the case of low signal to noise ratio. This is in accordance with the known results from the literature [1, 6, 11]. So in adaptive Kalman filtering, proper window length is chosen based on acceleration of the target and SNR. Through Kalman filter the target velocity and acceleration are estimated. Now the next unknown is the SNR which can be estimated as follows.

Proper window lengths for different accelerations with SNR as a parameter is shown in reference [1]. Based on the received signal SNR and the estimated acceleration from Kalman filter, if we select the window length particularly at low SNR estimated frequency has more error as shown in the simulated result. Hence, cubic spline interpolation technique is used for selecting the proper window length from the interpolated accelerations [1]. Spline interpolation equations are given in equations (5 to 12). Here the inputs are known window length for the estimated accelerations.

3.1 Choice of optimal window length

Window length which has less cumulative estimation error (cee) is taken as the optimal window length. Thus, cee is given as,

$$cee(k) = \frac{1}{k} \sum_{j=1}^k \frac{|v(j) - \hat{v}(j)|}{|v(j)|} \tag{13}$$

Cumulative error is estimated (cee) for the set of estimated velocity at a different window length for the estimated SNR.

B. Signal to noise ratio(SNR) estimation

$$SNR = 10 \log \frac{\hat{A}^2}{\hat{\sigma}_\mu^2(k)} \tag{14}$$

Where \hat{A}^2 - Estimated signal amplitude
 $\hat{\sigma}_\mu^2(k)$ - Variance of the estimation

IV RESULTS AND ANALYSIS

Here it is assumed that radar received the data every 2 milliseconds. Maximum number of samples for processing is 5000. (i.e.) the period of observation is $5000 \times 2 \times 10^{-3} = 10$ s. The velocity profile, while fast maneuvering, is assumed to change from 250 m/s to 116.6 m/s within 3.6 s. This gives the acceleration rate as -37.13 m/s^2 . Fig. 2 shows the interpolated window length corresponding to the interpolated accelerations at different SNR based on the reference [1].

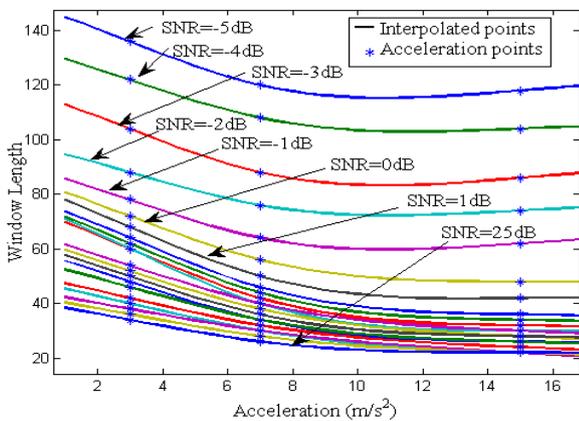


Fig.2 Interpolated acceleration versus window length for different SNR

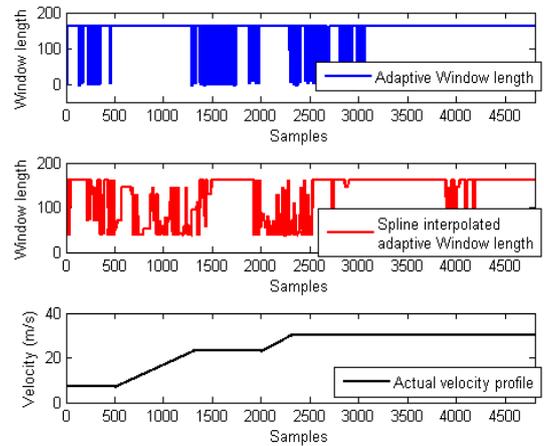


Fig. 3 Window length adaptation without and with interpolation for path 1 at 5dB SNR

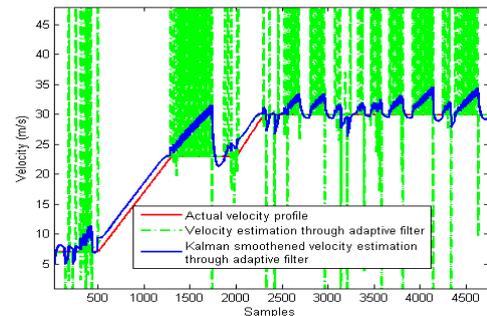


Fig. 4 Velocity estimation through adaptive window length for path 1 at 5dB SNR

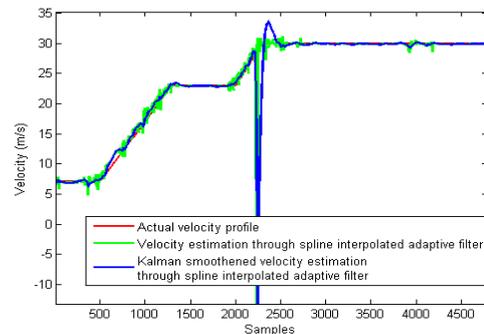


Fig. 5 Velocity estimation through interpolated adaptive window length for path 1 at 5dB SNR

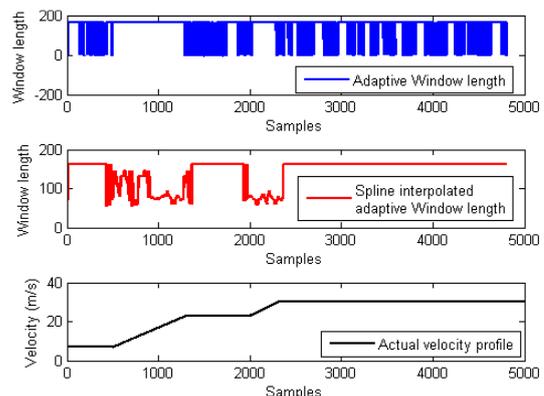


Fig. 6 Window length adaptation without and with interpolation for path 1 at 25dB SNR

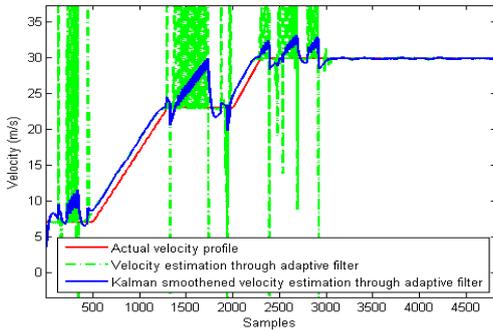


Fig. 7 Velocity estimation through adaptive window length for path 1 at 25dB SNR

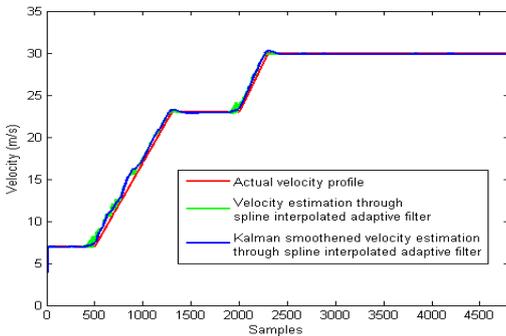


Fig. 8 Velocity estimation through interpolated adaptive window length for path 1 at 25dB SNR

Fig. 3 and 6 shows the adaptive window length without and with interpolation for the velocity profile with acceleration of path 1 at 5dB and 25dB SNR. It is observed that, velocity estimation is better with the interpolated adaptive window length as shown in Fig. 5 compared to Fig. 4. Better velocity estimation could be observed using Kalman smoothing. Same analysis are made at 25dB SNR as shown in Fig. 7 and 8. It is observed that interpolated adaptive Doppler filter estimated velocity with Kalman smoothing combination gave good estimation at low SNR.

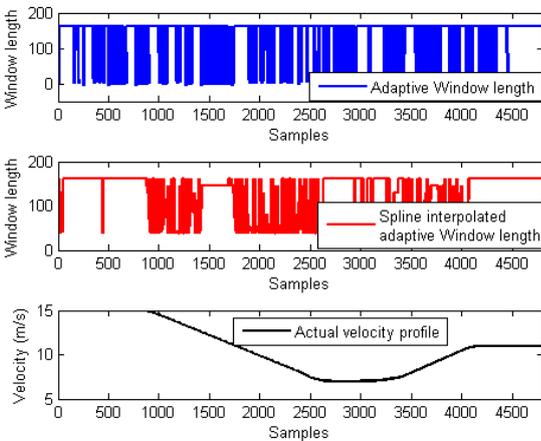


Fig. 9 Window length adaptation without and with interpolation for path 2 at 5dB SNR

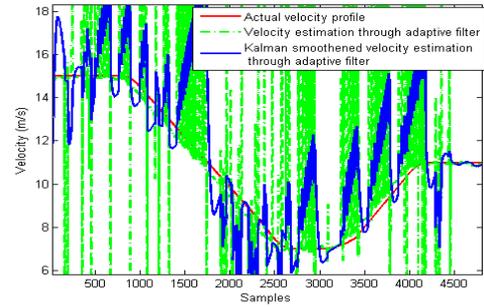


Fig. 10 Velocity estimation through adaptive window length for path 2 at 5dB SNR

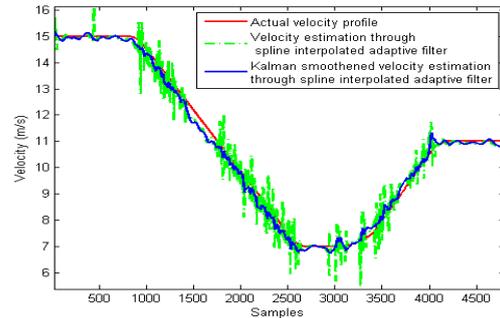


Fig. 11 Velocity estimation through interpolated adaptive window length for path 2 at 5dB SNR

Fig. 9 and 12 shows the adaptive window length without and with interpolation for the velocity profile with deceleration of path 2 at 5dB and 25dB SNR. Eventhough window length adaptation seems same with and without interpolation, performance of the velocity estimation holds the same way as shown in Fig. 10 to Fig. 14.

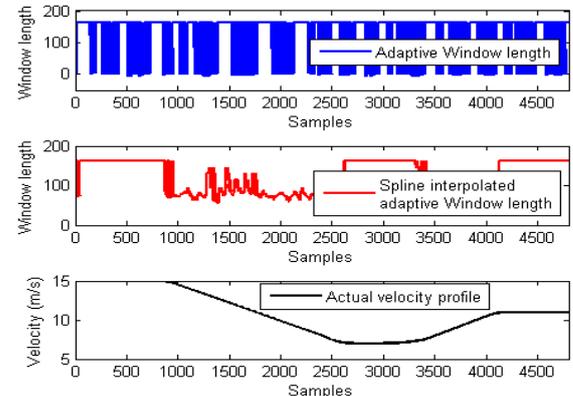


Fig. 12 Window length adaptation without and with interpolation for path 2 at 25dB SNR

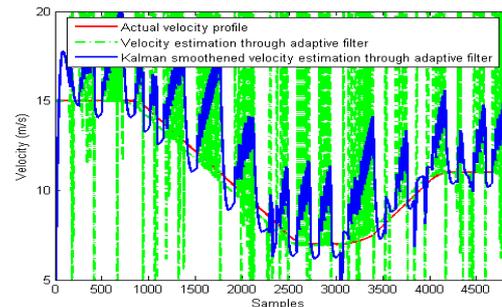


Fig. 13 Velocity estimation through adaptive window length for path 2 at 25dB SNR

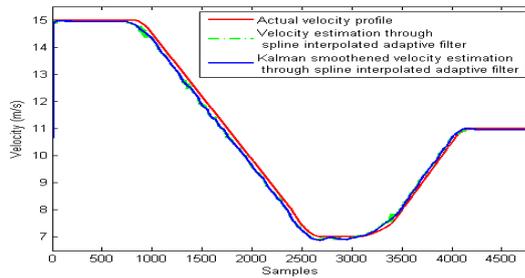


Fig. 14 Velocity estimation through interpolated adaptive window length for path 2 at 25dB SNR

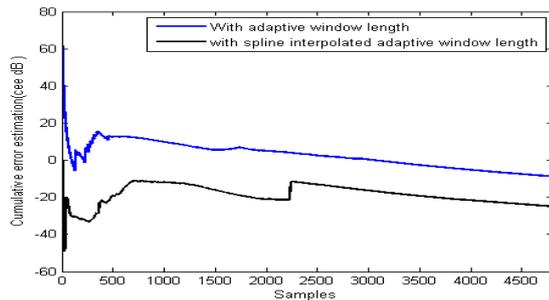


Fig. 15 Cumulative error for path 1 at 5 dB SNR

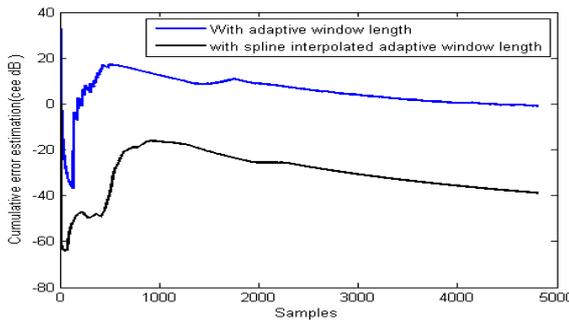


Fig. 16 Cumulative error for path 1 at 25 dB SNR

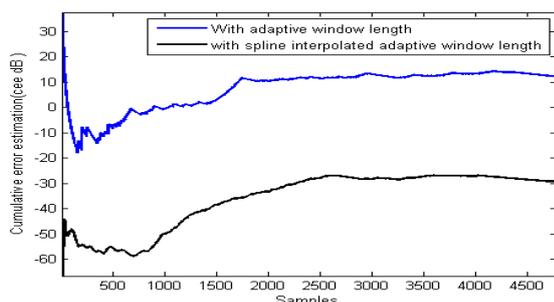


Fig. 17 Cumulative error at 5 dB for path 2

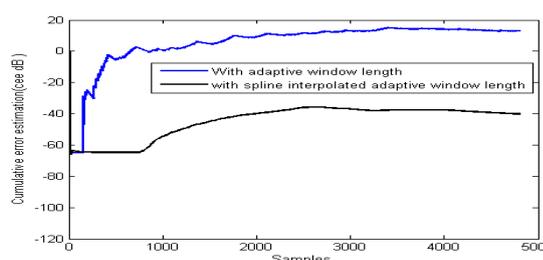


Fig. 18 Cumulative error at 25 dB for path 2

Cumulative error estimation(cee) plots are shown in Fig. 15 to Fig. 18 for paths 1 and 2 at 5dB and 25dB SNR. In all the plots it is observed that less cee in velocity estimation through interpolated adaptive Doppler filter compared to adaptive filter.

VI. CONCLUSION

Based on the result analysis it is clear that interpolated window length adaptation leads to the good velocity estimation compared to the earlier method. In general, track initialisation is the challenging task mostly accepted with allowable error. From the results it is observed that this error is reduced with spline interpolation. It is also observed that velocity estimation is better when the target is in acceleration compared to deceleration. Fighter aircraft with the velocity of 1.7 mach and the maneuverability of $9g$ to $-3.5g$ (88.2 m/s^2 to 34.3 m/s^2) may be tracked using our algorithm with reliable results. Other interpolation technique can also be used to compare the efficiency of the window length.

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