Selection Between Column and Row Decomposition in the Beamformer for Planar Array

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Abstract- **The planar array 2D beam formation can be divided into two sets of one-dimension beam formations [1]. This paper further studies the possibilities of decomposition on 2-D arrays. [1] Suggests, decomposition to row matrix (horizontal beamformer) and then to the column matrix(vertical beamformer). Alternatively the array can be decomposed by first selecting the column matrix(vertical beamformer) and then to the row (horizontal) beamformer. Both give same beam outputs. However the computation load involved in the process are different. Also apart from the 3 advantages listed in [1] and [2] there is huge saving in the computation load by decomposition.**

This paper a) derives the equivalence between the two ways of decomposition, and b) quantifies the computation advantage to aid in the selection of optimum decomposition process. The computation advantage is based on beam and array organization. In comparison with conventional processing, for an 'M' row x ' N' column planar array to form 'J' rows x ' K' column of beams, the computation load reduces to the fraction (M+K)/MK or (N+J)/NJ depending on whether we choose column or row processing first.

Key words: Digital Beamformer, Row beam former, column beam former

INTRODUCTION

The planar array beamforming is done by multiplication of complex weight for each beam with all elements of the array. This is repeated for the number of beams to get the final output.

Suppose we have a rectangular array with 'M' rows and 'N' column of elements as shown in Fig.1. Suppose we have to form J x K beams as shown in Fig. 2.

In the conventional way the array is considered as a single-dimensional matrix, X with 'MN'elements and each beam is formed individually by complex weight multiplication and summation on these. The beam formation algorithm is summarized as follows:

Y=CX

C= Coefficient matrix for the selected frequency to form JK beams. C is a JKx MN matrix. X is aMNx1 matrix..

Output Y is a JK x1 matrix

Hence to form JK beams we have to do, JK x MN complex multiplications in the conventional way.

Where,

M= no: of rows in the array

N= no: of columns in the array.

J = number of rows in the beam and

K = number of columns in beam.

No: of complex multiplications= MNJK (1)

DECOMPOSING THE BEAMFORMING PROCESS

The planar array 2D beam formation can be divided to two sets of one-dimension beam formations [1]. As per [1], the beam formation can be split to multiple steps.

Step1. We have to first use each row of the array, as a linear array and form the horizontal beams. .ie., in our case, from each of the 'M' rows, form 'K' beams. The output of this step is an MxK intermediate beam matrix as shown in Fig. 3.a.

Step 2. Next do the vertical beamforming. i.e, select each column of the intermediate beam matrix and form 'J' beams from each of the columns. This is shown in Fig. 3.b. This gives us JxK beams equivalent to that formed from the MxN planar array without decomposition to rows and columns.

As per [1] and [2], there are the following advantages with decomposition:

A. The decomposition process makes the design and incorporation of spatial windows much simpler.
B. The decomposition process leads to an efficient implementation in computing architectures based on the following two factors:

a) First, the number of the involved sensors for each of these line array beamformers is much less than the total number of sensors, of the planar array. This kind of decomposition process for the 2-D beamformer eliminates the need for very large memory and CPU's with very high throughput requirements in one board for real time system applications.
b) Secondly, all these line array beamformers can be executed in parallel, which allows their implementation in much simpler parallel architectures with simpler CPU's, which is a practical requirement for real time system application.

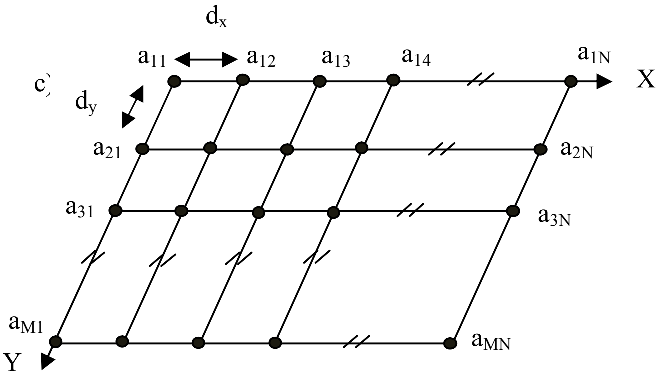

Figure.1 Array Organization: 'M' rows x 'N' Column of elements

C. Besides the advantage of the efficient implementation, the proposed decomposition approach makes incorporation of the adaptive schemes for planar arrays feasible.

METHODS IN DECOMPOSING

This paper further studies the possibilities of decomposition on 2-D arrays. [1] Suggests, decomposition to row matrix (horizontal beamformer) and then to the column matrix(vertical beamformer). Alternatively the array can be decomposed by first selecting the column matrix(vertical beamformer) and then to the row (horizontal) beamformer. This is shown in Fig.4.a and Fig.4.b.

Both give same beam outputs.

DERIVATION OF EQUIVALENCE

Let us derive the equivalence for forming the first beam B11.

Let B_{11} be oriented in (u1,v1) in beamspace. To form B_{11} the calculation is as follows.

$$B11 = a11.e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dx.u1+0.dy.v1]} + a12.e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dx.u1+0.dy.v1]} + \ldots\ldots + a1N.e^{-j\left(\frac{2\pi}{\lambda}\right)[(N-1).dx.u1+0.dy.v1]} + a21.e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dx.u1+1.dy.v1]} + a22.e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dx.u1+1.dy.v1]}$$

$$+\ldots\ldots + a2N.e^{-j\left(\frac{2\pi}{\lambda}\right)[(N-1).dx.u1+1.dy.v1]} + \ldots\ldots\ldots + aM1.e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dx.u1+(M-1).dy.v1]} + aM2.e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dx.u1+(M-1).dy.v1]} + \ldots\ldots + aMN.e^{-j\left(\frac{2\pi}{\lambda}\right)[(N-1).dx.u1+(M-1).dy.v1]} \ldots\ldots\ldots(2)$$

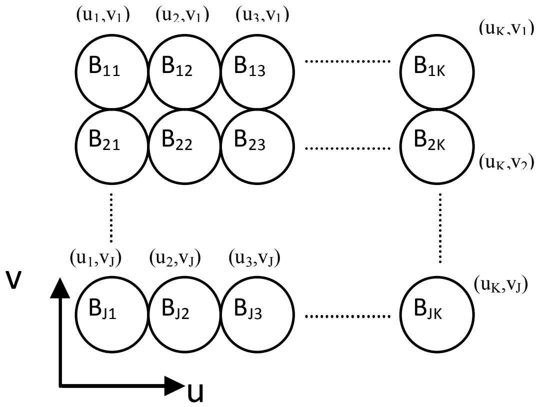

Figure.2 Beam Organization: 'J' rows x 'K' Columns

A. *COLUMN FISRT THEN ROW*

Rearranging (2),

$$B11 = e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dx.u1]} \left[a11.e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dy.v1]} + a21.e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dy.v1]} + \cdots + aM1.e^{-j\left(\frac{2\pi}{\lambda}\right)[(M-1).dy.v1]}\right]$$

$$+e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dx.u1]} \left[a12.e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dy.v1]} + a22.e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dy.v1]} + \cdots + aM2.e^{-j\left(\frac{2\pi}{\lambda}\right)[(M-1).dy.v1]}\right]$$

$$+\ldots\ldots\ldots$$

$$+e^{-j\left(\frac{2\pi}{\lambda}\right)[(N-1).dx.u1]} \left[a1N.e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dy.v1]} + a2N.e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dy.v1]} + \cdots + aMN.e^{-j\left(\frac{2\pi}{\lambda}\right)[(M-1).dy.v1]}\right] \ldots\ldots\ldots(3)$$

The terms in brackets are linear array beam formation equation for each of the column matrix and hence the above simplifies to,

$$B11 = e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dx.u1]}.IB_{1v1} + e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dx.u1]}.\ IB_{2v1} + \ldots\ldots + e^{-j\left(\frac{2\pi}{\lambda}\right)[(N-1).dx.u1]}.IB_{Nv1} \ldots\ldots\ldots(4)$$

Where IB_{1v1} is the intermediate beam formed in v1 direction from 1st column and IB_{Nv1} is the intermediate beam in v1 direction from Nth column. Hence the above simplifies to,

$$B_{11} = IB_{1v1}.\ e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dx.u1]} + IB_{2v1}.\ e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dx.u1]} + \ldots\ldots + IB_{Nv1}.\ e^{-j\left(\frac{2\pi}{\lambda}\right)[(N-1).dx.u1]} \ldots\ldots\ldots(5)$$

which is the beam formation equation to form beam in u1 direction from a row matrix of beams .Thus we get the (u1, v1) beam by first using the column matrix (to form beam to v1) and then the row matrix(to form beam to u1) .

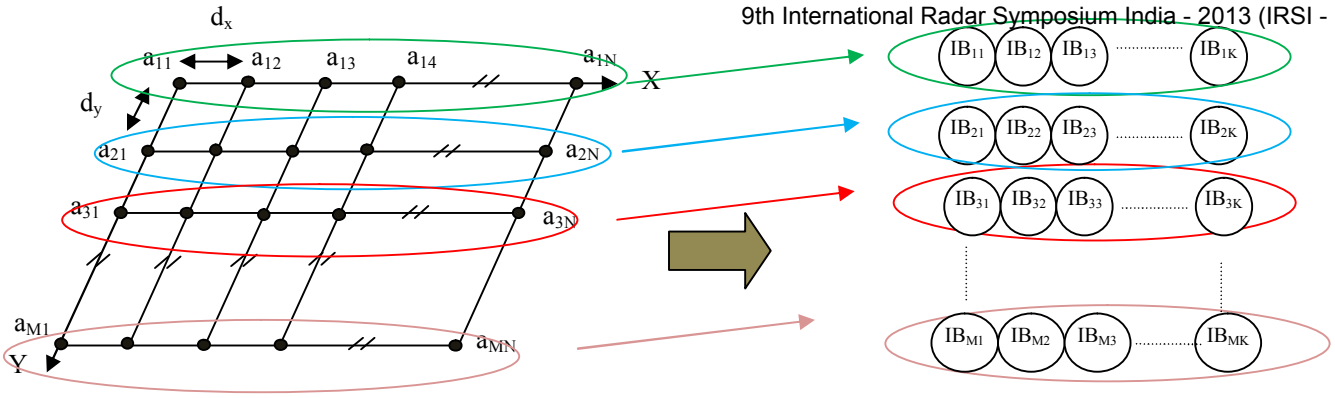

Figure.3.a. First each row of the MxN array is used as a linear array and 'K' horizontal beams are formed from each row. The output of this step is a MxK intermediate beam matrix

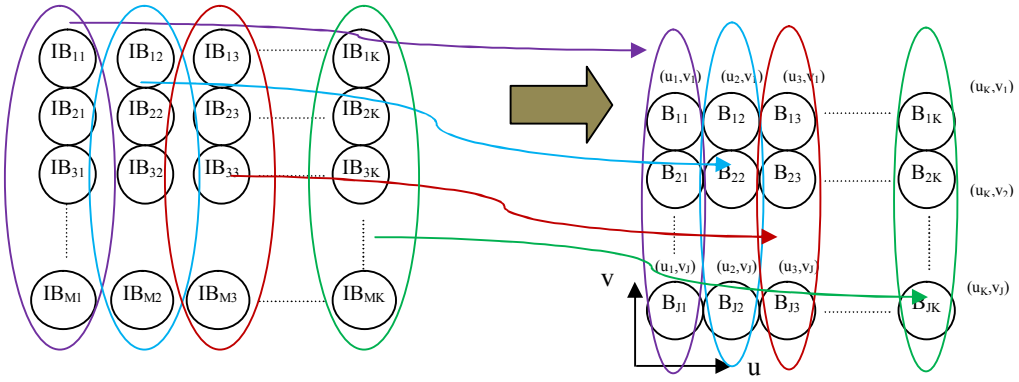

Figure.3.b. Vertical beamforming. Each column of the MxK intermediate beam matrix is used to form 'J' beams. This gives us JxK beams equivalent to that formed from the MxN planar array without decomposition to rows and columns.

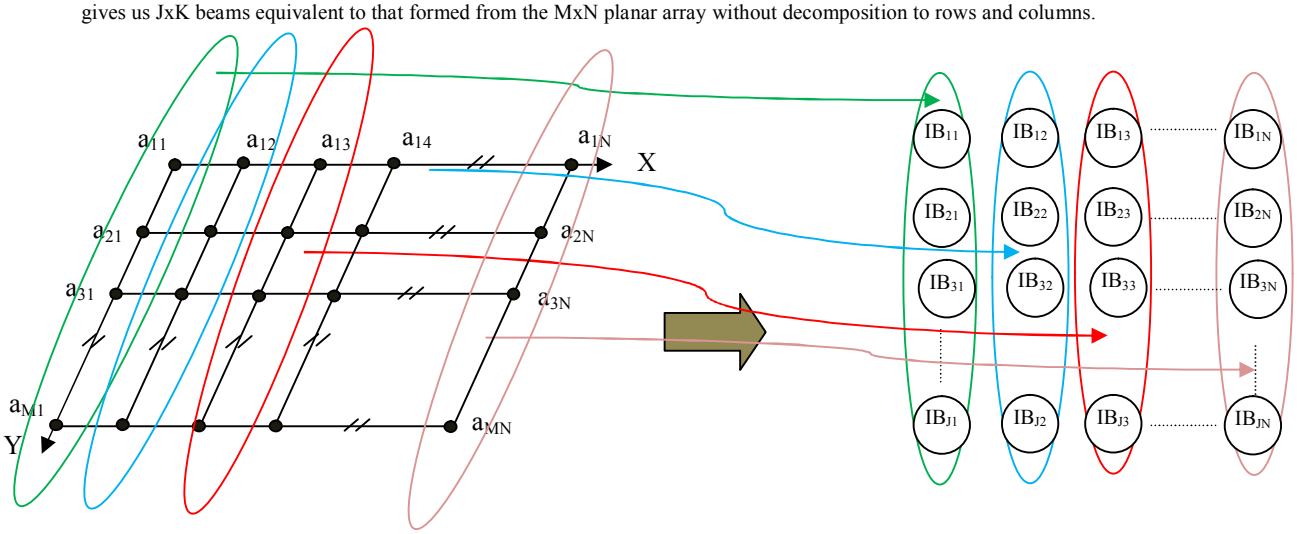

Figure.4.a. First each column of the MxN array is used as a linear array and 'J' vertical beams are formed from each column. The output of this step is a JxN intermediate beam matrix.

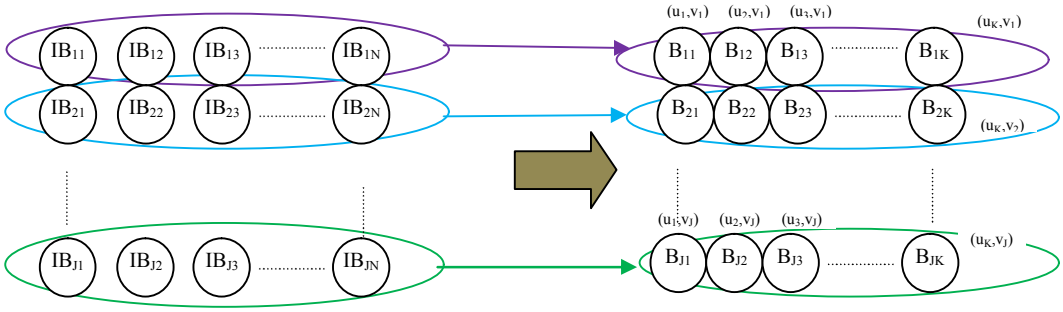

Figure.4.b. Horizontal beamforming. Each row of the JxN intermediate beam matrix is used to form 'K' beams. This gives us JxK beams equivalent to that formed from the MxN planar array without decomposition to rows and columns.

B. ROW FIRST THEN COLUMN

We can rearrange (2) alternatively as , B_{11}=

$$e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dy.v1]}\left[a11.e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dx.u1]} + a12.e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dx.u1]} + \cdots + a1N.e^{-j\left(\frac{2\pi}{\lambda}\right)[(N-1).dx.u1]}\right]$$
$$+e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dy.v1]}\left[a21.e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dx.u1]} + a22.e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dx.u1]} + \cdots + a2N.e^{-j\left(\frac{2\pi}{\lambda}\right)[(N-1).dx.u1]}\right]$$
$$+\ldots\ldots$$
$$+e^{-j\left(\frac{2\pi}{\lambda}\right)[(M-1).dy.v1]}\left[aM1.e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dx.u1]} + aM2.e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dx.u1]} + \cdots + aMN.e^{-j\left(\frac{2\pi}{\lambda}\right)[(N-1).dx.u1]}\right] \ldots\ldots (6)$$

The terms in brackets are linear array beam formation equation for each of the row matrix and hence the above simplifies to,

$$B11 = e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dy.v1]}.IB_{1u1} + e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dy.v1]}.IB_{2u1} + \ldots\ldots + e^{-j\left(\frac{2\pi}{\lambda}\right)[(M-1).dy.v1]}.IB_{Mu1} \ldots\ldots (7)$$

Where IB_{1u1} is the intermediate beam formed in u1 direction from 1st row and IB_{Mu1} is the intermediate beam in u1 direction from Mth row. . Hence the above simplifies to,

$$B_{11} = IB_{1u1}.e^{-j\left(\frac{2\pi}{\lambda}\right)[0.dy.v1]} + IB_{2u1}.e^{-j\left(\frac{2\pi}{\lambda}\right)[1.dy.v1]} + \ldots\ldots + IB_{Mu1}.e^{-j\left(\frac{2\pi}{\lambda}\right)[(N-1).dy.v1]} \ldots\ldots (8)$$

which is the beam formation equation to form beam in v1 direction from a column matrix of beams .Thus we get the (u1, v1) beam by first using the row matrix (to form beam to u1) and then the column matrix(to form beam to v1) .

The same is applicable to other beams B_{12} to B_{JK}.

COMPUTATION LOAD CALCULATION AND DECOMPOSITION METHOD SELECTION

However the computation load involved in the process are different. Also apart from the 3 advantages listed in [1] and [2] there is huge saving in the computation load by decomposition. This paper a) derives the equivalence between the two ways of decomposition as done in (5) and (8), and b) quantifies the computation advantage to aid in selection of decomposition process. The selection has to be optimized based on beam and array organization which are detailed subsequently.

A COLUMN BEAM FORMER FIRST THEN ROW

Here we have to do column processing first. i.e., first form intermediate vertical beams using column wise processing of elements. To form vertical beams from each column we need 'M' complex multiplication for each column for each beam. ie, we need a total of 'MJ ' complex multiplications for each column to form 'J' intermediate beams.

With a total of 'N' columns we need MNJ multiplications. Now we have 'N' columns of 'J' vertical beams each. From this do row processing (horizontal beam forming) to form final beams. To form the first row of 'K' horizontal final beams we need, NK multiplications. So to form all 'J' rows we need NKJ multiplications. Hence the total number of multiplications needed are MNJ+NKJ= NJ(M+K)

Multiplications with column first= NJ(M+K) (9)

B COMPARISON OF CONVENTIONAL AND COLUMN BEAM FORMER FIRST METHOD

From (1) and (9), the ratio of computations (complex multiplications) needed for the processing = NJ(M+K)/ MNJK= (M+K)/MK

The reduced load with column first=(M+K)/MK (10)

C ROW BEAM FORMER FIRST THEN COLUMN

Here we have to do row processing first. i.e., first form intermediate horizontal beams using row wise processing of elements. To form horizontal beams from each row we need 'N' complex multiplication for each row for each beam. ie, we need a total of 'NK ' complex multiplications for each row to form 'K' intermediate beams.

With a total of 'M' rows, we need MNK multiplications. Now we have 'M' row of 'K' horizontal beams each. From these do column processing (vertical beam forming) to form final beams. To form the first column of 'J' vertical final beams we need, MJ multiplications. So to form all 'K' columns we need MJK multiplications. Hence the total number of multiplications needed are MNK+MJK= MK(N+J)

Multiplications with row first= MK(N+J) (11)

D *COMPARISON OF CONVENTIONAL AND ROW BEAM FORMER FIRST METHOD*

From (1) and (11) the ratio of computations (complex multiplications) needed for the processing = MK(N+J)/MNJK=(N+J)/NJ

The reduced load with row first= (N+J)/NJ (12)

EXAMPLES

This is quite a good savings. Let us work out some examples.

1 EXAMPLE-1

Assume a radar with 192 elements organized as 16* 12 array.
Let the beam organization be 3*3.
M=16, N=12, J=3, K=3;
The loads needed are only 39.5% and 41.7% respectively and for method 1 and method2 using (10) and (12). For this array and beam organization, column decomposition first is the optimum method.

2 EXAMPLE-2

Assume a radar with 48 elements organized as 24* 2 array.
Let the beam organization be 8*2.
M=24, N=2, J=8, K=2;
The loads needed are only 54.1% and 62.5% respectively and for method 1 and method2 using (10) and (12). For this array and beam organization, column decomposition first is the optimum method.

3 EXAMPLE-3

Assume a radar with 960 elements organized as 60* 16 array. Let the beam organization be 12 *2.
M=60, N=16, J=12, K=2;
The loads needed are only 51.6% and 14.5% respectively and for method 1 and method2. The computation load needed is only 14.5% for row beamformer first compared to the conventional way. For this array and beam organization, row decomposition first is the optimum method.

CONCLUSION

The decomposition of 2-D planar array into two linear arrays can be done to aid easy beam formation. The decomposition can be done in two ways:

1. Column beamformer first then row beam former.
2. Row beamformer first then column beamformer.

In general, the computation load reduces to the fraction (M+K)/MK or (N+J)/NJ depending on whether we choose column or row processing first for an 'M' row x 'N' column array to form 'J' rows x 'K' column of beams. This is a good saving in comparison to conventional method. The selection of whether to do row or column decomposition first depends on the array and beam organization. The efficiency of the said method is better for certain beam and array organizations.

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