Design of Inverse Moving Target Indicator (IMTI) algorithm for arbitrary filter weights

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Abstract:

This paper discusses a novel approach of computing IMTI coefficients. The problem of determining IMTI coefficients is formulated as linear transformation and the computation exploits the symmetry, sparseness and multiplication property of the matrices involved. This brings down the computational complexity.

I. Introduction

Airborne pulse Doppler radar performs moving notch MTI filter to remove main lobe clutter (MLC) [1]. This operation is followed by CFAR process, 1D CFAR along range axis and another 1D along Doppler axis. The MTI operation introduces modulation in frequency domain along slow time. The modulation introduced by MTI operation has to be compensated before performing CFAR in Doppler direction. This is achieved through Inverse MTI (IMTI). The IMTI compensation is performed in frequency domain.

This paper has five sections. Section I discusses on the introduction, Section II describes problem formulation, section III brings out the computation method adopted in performance optimization. Results are shown in Section IV and Conclusions are drawn in section V.

II. Problem formulation

The objective of IMTI is to compensate the modulation introduced by the MTI filter. MTI operation is generally a moving average filter (MA filter) with arbitrary length and filter weights. The primary objective of MTI filter is to cancel main lobe clutter (MLC) before coherent processing, using Discrete Fourier Transform (DFT). The operation by MTI filter is modeled as linear transformation by convolution matrix (*A*) on a data vector *x*. The processed data vector (*y*) is given as

$$y = Ax$$

To prevail the effects of FFT side lobes windowing is done on data, prior to FFT [Haris]. The output data vector (*y*) undergoes windowed FFT to produce ($\hat{y}$). surmount

$$\hat{y} = DWAx$$

In order to compute the IMTI weights, covariance matrix of the output data is computed.

$$E\{\hat{y}\hat{y}^H\} = D * W * A * A^H * W^H * D^H$$

IMTI weights can be calculated as follows

$$IMTI_{coeff.} = \sqrt{\frac{1}{Diag_{cov}}} \qquad (1)$$

Where,

$$Diag_{cov} = Dioganal\ elements\ of\ \mathrm{cov}ariance\ matrix\ R$$

$$R = E\{\hat{y}\hat{y}^H\} \qquad (2)$$

Where,

D = DFT matrix, (N*N)

W = Window matrix, (N-M+1 * N-M+1)

A = Convolution matrix, (N-M+1 * N)

M = length of MTI coefficients

N = Number of Integration pulses

Computation of R matrix is resource intensive, it involves three matrix multiplication operation with each matrix of dimension N. This requires $3N^3$ multiplications and $3N^3$ summation operations. And one N dimension matrix inverse operation.

III. Computational method

We have optimized the computation of IMTI coefficient by avoiding matrix multiplication to the minimum possible and also reduced the number of matrix operations by utilizing the inherent properties of each matrix. Further single step computation of R as shown in equation 2 is broken into multiple steps; this is done to bring optimization at each step. The entire process is explained as follows

STEP I: AA^H

Convolution matrix C is a Circulant matrix of dimension N*N. Circulant matrix is a sparse matrix. The sparseness of C depends on difference in length of MTI filter coefficients M and N. A sub matrix A derived from C, is of dimension N-M+1*N. Matrix A is sparse as generally M<<N. To compute AA^H efficiently we exploit the Circulant property and sparseness of matrix A. This is explained using following example matrix P.

- A circulant matrix P is assumed to be of size N-M+1 * N, which is a submatrix of the circulant matrix C of size N*N.
- Vector b is defined of size 1*3, and is the base vector which is circularly shifted to form P.
- P^H is formed by taking Hermitian of matrix P

When we analyse the matrix product PP^H interesting properties can be identified.

- Due to N>>M the matrix PP^H is sparse.
- M^{th} row repeats till $(N-2M)^{th}$ row, being circularly shifted at each row
- Submatrix of PP^H of dimension M*2M-1 is a Teoplitz matrix.
- Among first M rows only 1^{st} row is unique, which is the only row to be computed.
- Last M rows from N-M+1 to N-2M can be obtained by flipping the first M row from 1:M

Thus the operation of AA^H can be avoided and its purpose can be sufficed by the submatrix S1 of AA^H of dimension M*2M-1. S1 consists of 1^{st} M rows of AA^H matrix and 2M-1 column. S1 is sufficient to replace the use of AA^H in equation 2.

$$b = [b1 \quad b2 \quad b3], n = 3(\text{length of } b) \quad (3)$$

$$P = \begin{bmatrix} b1 & b2 & b3 & 0 & 0 & 0 & 0 \\ 0 & b1 & b2 & b3 & 0 & 0 & 0 \\ 0 & 0 & b1 & b2 & b3 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & b1 & b2 & b3 \end{bmatrix}$$

(4)

$$P^H = \begin{bmatrix} b1 & 0 & 0 & \cdots & 0 \\ b2 & b1 & 0 & \cdots & 0 \\ b3 & b2 & b1 & & 0 \\ 0 & b3 & b2 & \ddots & 0 \\ 0 & 0 & b3 & \ddots & b1 \\ 0 & 0 & 0 & \ddots & b2 \\ 0 & 0 & 0 & \cdots & b3 \end{bmatrix}$$

(5)

$$PP^H =$$

$$\begin{bmatrix} \sum_{i=1}^{n} b_i^2 & \sum_{i=1}^{n-1} b_{i+1}b_i & \sum_{i=1}^{n-2} b_{i+2}b_i & 0 & 0 & 0 & & \cdots & 0 \\ \sum_{i=1}^{n-1} b_i b_{i+1} & \sum_{i=1}^{n} b_i^2 & \sum_{i=1}^{n-1} b_{i+1}b_i & \sum_{i=1}^{n-2} b_{i+2}b_i & 0 & 0 & & \cdots & 0 \\ \sum_{i=1}^{n-2} b_i b_{i+2} & \sum_{i=1}^{n-1} b_i b_{i+1} & \sum_{i=1}^{n} b_i^2 & \sum_{i=1}^{n-1} b_{i+1}b_i & \sum_{i=1}^{n-2} b_{i+2}b_i & 0 & 0 & \cdots & 0 \\ 0 & \sum_{i=1}^{n-2} b_i b_{i+2} & \sum_{i=1}^{n-1} b_i b_{i+1} & \sum_{i=1}^{n} b_i^2 & \sum_{i=1}^{n-1} b_{i+1}b_i & \sum_{i=1}^{n-2} b_{i+2}b_i & 0 & \cdots & 0 \\ & & & & & & & & 0 \\ \vdots & & \ddots & \ddots & \ddots & \ddots & \ddots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & 0 & \sum_{i=1}^{n-2} b_i b_{i+2} & \sum_{i=1}^{n-1} b_i b_{i+1} & \sum_{i=1}^{n} b_i^2 \end{bmatrix}$$

(6)

STEP II: WAA^HW^H => $WS1W^H$

Here W and W^H is multiplied with AA^H in an optimized way, where W is a diagonal matrix of size N-M+1. The diagonal elements of matrix W are the coefficients of the window function. Window function can be Rectangular, Hamming, Henning etc. Length of window coefficients is N-M+1. But this operation can be optimized due to the symmetry property of Window coefficients. Also the uniqueness of two diagonal matrix multiplication which is shown below.

$$\text{if } C = \begin{bmatrix} c0 & 0 & 0 \\ 0 & c1 & 0 \\ 0 & 0 & c0 \end{bmatrix} \tag{7}$$

$$Test = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \tag{8}$$

then, $C * Test * C^H$ is

$$\begin{array}{cc} & \begin{array}{ccc} c0 & c1 & c2 \\ \downarrow & \downarrow & \downarrow \end{array} \\ \begin{array}{c} c0 \rightarrow \\ c1 \rightarrow \\ c2 \rightarrow \end{array} & \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \end{array} \tag{9}$$

The resultant matrix shows that each row and each column is scaled by the corresponding diagonal element of the diagonal matrix. Due to symmetry of A and W, WAA^HW^H matrix is also symmetric. Thus only half the dimension is required to be computed and rest half can be replaced by the mirror image of first half. This operation is further optimized as follows,

- The matrix multiplication AA^HW^H can be replaced by vector product of each row of AA^H with respective row of W^H.

Each row of S1 is vector multiplied with the first 2M-1 coefficients of Window function. In order to create (N-M+1)/2*N-M+1 matrix, M+1 row to N/2 row are computed using M^{th} row of S1. This is done by applying sliding window technique on N-M+1 coefficients of Window function. The sliding window is the M^{th} row of S1 matrix. The second half of the matrix is computed by flipping the first half i.e (N-M+1)/2*N-M+1 matrix. This is done by rearranging the rows as (N-M+1)/2 to 1 then rearranging column as N-M+1 to 1. First half is appended with the conjugate of second half to form the N-M+1 * N-M+1 matrix.

STEP III: $DWAA^HW^HD^H$

In the last step to find the IMTI coefficients is to perform matrix multiplication with DFT matrix D and its Hermmitian matrix D^H. D matrix size is N*N so in order to perform $WAA^HW^HD^H$ matrix multiplication the matrix so formed in step II is appended with zeros to make its size from N-M+1*N-M+1 to N*N. Temporary matrix T is formed which is

$$T = WAA^HW^HD^H \tag{10}$$

STEP IV: IMTI_coefficients

The IMTI coefficients are inverse of the vector formed by diagonal elements of the matrix D*T. The matrix multiplication operation is avoided and directly the diagonal elements are computed as follows

- Vector product of each row of matrix D with each row of the transpose of matrix T is taken.
- Sum of all rows is taken

The vector formed by the sum of all rows is the required diagonal vector.

Finally, as per equation 1 IMTI coefficients are generated.

IV. Results

The results of the optimized algorithm are shown as under.

AWGN complex noise was generated with noise variance of -104dBm. N=256, M = 3 and h = [1 -2 1]. Hamming window function was used.

Figure 1 shows power of input noise signal in dB scale Vs samples.

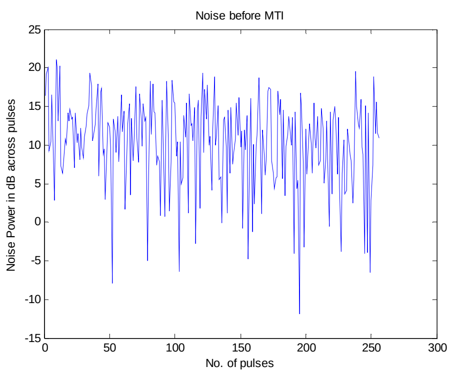

Figure 1

Figure 2 shows output of MTI and Coherent processing i.e. output of MTI in frequency domain. The arc shape represents the MTI modulation.

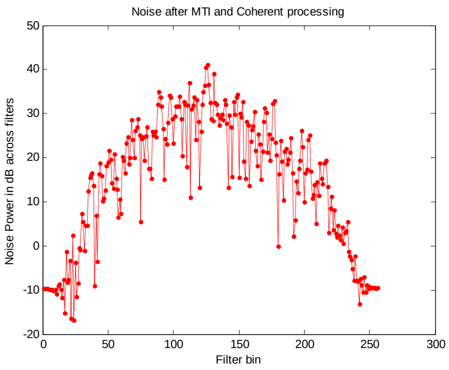

Figure 2

Figure 3 represents the frequency response of IMTI coefficients.

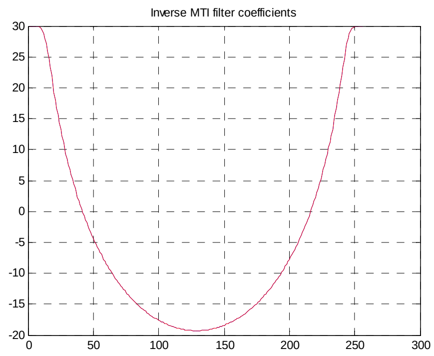

Figure 3

Figure 4 represents the frequency response obtained by multiplying the IMTI coefficients with frequency response of MTI output. The result shows that IMTI is able to demodulate the signal to its original power level.

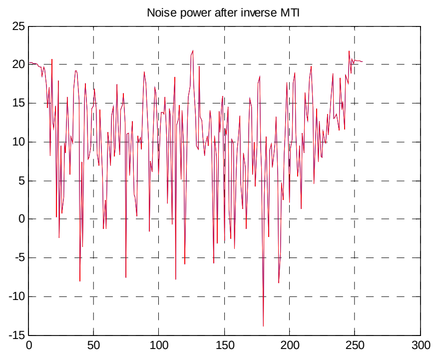

Figure 4

V. Conclusion

The efficient computation method has resulted in the reduction of computation complexity and load.
The simulation results proved its effectiveness in nullifying the effect of MTI modulation on signal.

VI. Refrences

1. *G. W. Stimson, Introduction to Airborne radar, Scitech publication. [crossref]*

Author Info

Shri. Narasimhan RS is currently working in Electronics & Radar Development Establishment, Bangalore as Scientist - D. He obtained his B.E in Electronics in the year 2002 from Visvesvaraya Technological University and M.E degree in system sciences and automation from Indian Institute of Science (IISc) Bangalore in 2009. His area of interest are Radar signal & data processing.

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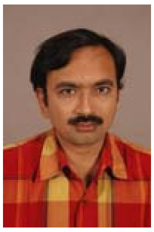

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